

# ACLE conference

Monday September the 27<sup>th</sup> 2004

## MLATS

### Mathematics Learning and Teaching for Success

Plan of the workshop

Introduction

Aims of the session

Dot cards activity

- ◆ Participate
- ◆ Share
- ◆ Reflect

What can we learn from this story about helping children learn mathematics successfully?

1. About teachers
  - ◆ teacher assumptions
  - ◆ teacher knowledge
  - ◆ and what they value
  - ◆ getting to know children and what they know
2. About mathematics and numeracy
  - ◆ what puts children "at risk"
  - ◆ assessment

I will weave information about mathematics and numeracy from current research, and implications of this for teachers and schools, into this structure.

To conclude we will briefly consider brain based learning and what it means for the teaching and learning of mathematics.

## **Introduce self – what I have been involved in**

- ◆ Early childhood trained - worked with children of all ages from K-7
- ◆ Became passionate about the T&L of ma in the 1980's - this passion has stayed with me
- ◆ Worked as a lecturer with pre-service teachers
- ◆ Co-developed MLATS course with ML through the 1990's - now run the MLATS company where we run PD for teachers, parents and other educators
- ◆ latest working on intervention program - what does that look like??
- ◆ who and what has impacted on my learning?
  - The children I have worked with over the years - I have learnt more from the children that I "taught" than I ever learnt at school or at Teachers college
  - BLIPS
  - LLIMY
  - The teachers I have worked with over the years
  - Helen Pengelly, Sue Willis, Geoff White, Paul Swan, Doug Clarke, van de Walle...
  - Theories of constructivism, Vygotsky and his zone of proximal development, research into brain based learning

## **Aims of this workshop**

- ◆ Challenge you to think about what you do and why you do it ...
- ◆ Encourage you to reflect on your own life story to this point - how can you use your own experiences to encourage and support others
- ◆ To consider a range of themes from recent research and what this means for the teaching and learning of mathematics and the development of numeracy
- ◆ For you to leave the workshop with a renewed commitment to really get to know the children you teach

## Dot cards activity

### Participate

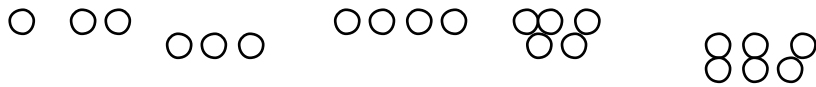
- ◆ How many dot would you need to make the dot cards 1-10?
- ◆ How many dot would you need to make the dot cards 1-100?
- ◆ How many dot would you need to make the dot cards 1-1000?
- ◆ How many dot would you need to make the dot cards 1-any number?
- ◆ What if you didn't start at 1?
- ◆ What if you just made odd numbers? Or even numbers?
  
- ◆ How many dots would you need to make a set of 1-10 dot cards for every one in the class?
- ◆ If one box of dots has 500 dots, how many boxes would we need to buy?
- ◆ What size should we make the cards to stick the dots on to and why? What would be the quickest way to make sure everyone gets the right number of dots and why?
- ◆ If you imagined there was a ten frame on each card, how many different ways would there be to arrange the dots?

### Share

- ◆ Strategies
- ◆ Make the comment that all of these strategies are valid
- ◆ Not all efficient past 100; mathematics is an ongoing exploration of pattern to help us solve problems with efficiency and accuracy

## OHT of strategies for solving dot cards question

1. Using counters to represent dots OR drawing all



2. Use a calculator

3. Adding all in sequence

Either  $1+2+3+4+5+6+7+8+9+10$  OR

$10+9+8+7+6+5+4+3+2+1$

4. "Chunking" in different ways

$1+2+3+4+5=X$      $6+7+8+9+10=Y$      $X+Y=$

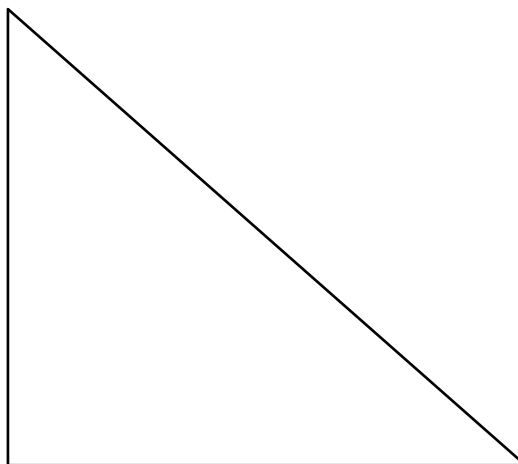
5. Pairing to make 10

$10$      $9+1$      $8+2$      $7+3$      $6+4$      $5$   
 $5 \times 10 + 5$

6. Pairing to make 11

$10+1$      $9+2$      $8+3$      $7+4$      $6+5$   
 $5 \times 11$

7.If we arranged the dots in a triangle, we would have rows from having one dot to having ten dots.



If we took the first 5 rows and rotated them around to fit onto the other rows we would have a rectangle of dots, with 5 rows of 11 dots each. A total of 55 dots.

One group continued this line of investigation and theorised that if they were to find the number of dots for any number starting at 1, they could say  $\frac{1}{2}$  base x height, where the base was the number of dots cards ie 10 and the height was the number of dots plus 1 ie 10+1.

$\frac{1}{2}$  (no of dot cards) x height (number of dot cards plus 1) = total number of dots

7.To add any series of consecutive numbers (n) (starting at 1)

$$n+1 \times 1/2n$$

$$n(n+1) \div 2$$

$$n \times 1/2n+n$$

$$\frac{n(n+1)}{2}$$

## Reflection:

- ◆ What previous knowledge did you draw upon to work through this question? (The extent to which you can draw on this knowledge will depend to a great extent on whether you really learnt it in the first place - there is continuing debate over what constitutes learning)
- ◆ To what extent was your participation influenced by your attitude towards mathematics or your ideas about yourself as a mathematician?

## What can we learn from this story about L&T Mathematics?

### *Teacher assumptions*

I had made assumptions about Natalie, Tim and Nick that were inaccurate.

When we assume that students "know" or "can do" something when in actual fact they only "know" it in a very narrow way or in one specific context, or if we assume that they "don't know" or "can't do" something, it can lead to a number of problems. When, for example, a teacher observes correct responses and assumes that the student has understood a concept, the teacher is likely to move them onto new learning experiences. However, if the mental images and relationships to which the student is trying to connect new knowledge are poorly developed, future constructs are weak and confused, and misconceptions may arise (Steinle & Stacey, 1998). Secondly, if students can execute a procedure but do not have understanding, it is unlikely that they will be able to use this knowledge when it is called upon in a new situation (Willis, 2000; Baturu, 1998).

We often assume because a child can do something they can do something else, or because they can't do something they can't do something else. Sue Willis, research for First Steps children who did not know rote sequence to ten but who could subitise to 6 or 7.

Thomas (1996) portrays the growth in constructing knowledge about the number system as "unpredictable, involving regressions as well as progressions and being, more often than not, non-linear" (p. 93). Whilst it remains important for teachers to know about broad stages of development, it is also important to stress that not all children develop in the same way or at the same time, nor can we assume that "the same early indicators and sequencing are equally appropriate for all children" (Willis, 2000, p. 32). Early Years Numeracy Interview Data - collated yr 1 data 90 different profiles of children in relation to what they knew

If we don't recognize this, it is easy to label children who learn different things at different times in different order to their peers as "behind" or as being "at risk (of not meeting the national Numeracy goal). It is also easy to move these students through learning experiences that, instead of building on their existing knowledge, may actually undermine their current knowledge, and hinder the development of further knowledge. These findings by Willis (2000, p. 32) indicate that "differences between children in their learning of mathematics can neither be explained nor accommodated by variations in the pace at which they develop certain mathematical concepts. Rather, there may be differences in the very nature and sequence of their development of mathematical ideas."

Educators are increasingly aware of the extent to which the official curriculum and much educational practice has tended to favour some learners and disadvantage others." (SACSA Framework pg 18)

### **Reflection:**

Think about the children you work with. What assumptions have you made about them? How can you check whether your assumptions are accurate or not?

Which students are favored by your school curriculum and pedagogy and which are disadvantaged?

### ***Teacher knowledge of mathematics***

I didn't know the mathematics; therefore I didn't know the questions to ask. I told Tim to read a book. I didn't know the patterns that could be explored through this simple question. I didn't know all the strategies that people would use to work it out. I didn't know the relationships between the different strands and sub strands of mathematics. Over the years I have learnt so much about mathematics as a whole, but I am still learning new connections and relationships.

In 2001 I was involved in a National Research project where we investigated the effectiveness of the Base Ten game in developing children's understandings of the number system. This project revealed many interesting insights, including that:

- Teachers needed to develop their own knowledge of the base ten number system before they could help students learn;

- Once teachers had developed their own relational understanding of the number system, they were better able to:
  - discover what each student already knew about base ten;
  - diagnose any misconceptions that a student may have developed;
  - offer learning activities that enabled students to build their knowledge;
  - adapt learning activities to meet the individual learning needs of the diversity of students in the class

*Did you know?*

...that there has been significant research into algorithms and their effect on children' ability to think mathematically.

Jamie (2<sup>nd</sup> grade girl), reported in Narode, Board, & Davenport (1993):

- [Early in the school year] Jamie added 19 and 26 mentally ("I know I have 30 because I have a group of ten and two more tens. Then if I take 1 from the 6 and give it to the 9, I'll have another group of 10. That leaves five left, so the answer is 45").
- [After five months of school and work with conventional algorithms] Jamie attempted to add 34 and 99 by beginning to group the 9 tens and 3 tens, then stops and says, "Oh, I have to add the ones first." She then grouped the units, and traded for a ten to solve the problem".
- [In the last month of the school year] Asked about the possibility of solving the problem by adding the tens first, Jamie emphatically stated, "No, you never add the tens first". Instead, she suggested that another way to solve the problem might be to know the answer from memory. Finally, she was confronted with her own invented strategy as a strategy "someone used" to add  $49 + 19$  (I think of  $50 + 19$  and then subtract one to get 68). When asked if she thought this method might work, she replied "If you know that way, it's okay, but it's much, much better to just add the ones first".

Research clearly suggests that if children are introduced to algorithms too early it will actually hinder their mathematical thinking.

Kamii & Dominick (1998): Twice as many third graders who had not been taught written algorithms as those who had, successfully answered  $6 + 53 + 185$  (50% compared with 25%). Interestingly, the answers of the "no-algorithms group" were all in the range 221-284, while the others ranged from 29 to 838.

What does the research into young children and algorithms say?

**OHT Dangers inherent in written algorithms (adapted from Kamii, 1998; McIntosh, 1998; Usiskin, 1998)**



- They do not correspond to the ways in which people tend to think about numbers (The belief that algorithms train the mind has little basis)
- They encourage children to give up their own thinking, leading to a loss of "ownership of ideas"
- They tend to "unteach" place value, thereby preventing children from developing number sense
- They tend to make children dependent on the spatial arrangement of digits (or paper and pencil)
- They tend to lead to blind acceptance of results, and overzealous applications (eg 21-19 in vertical form)
- Adults use formal written computation for less than 25% of their calculations (it has become increasingly unusual for standard written algorithms to be used anywhere except in the mathematics classroom)

Trial "How did you do it?" with your children.

### **When should children meet the conventional algorithms?**

Some scholars (e.g., Alistair McIntosh) believe that primary children should never be taught written algorithms, arguing that they get in the way of the most important computation method: mental computation.

"The learner should never be told directly how to perform any operation in arithmetic. . . . Nothing gives scholars so much confidence in their own powers and stimulates them so much to use their own efforts as to allow them to pursue their own methods and to encourage them in them". (Colburn, 1912, p. 463)

Others argue for delaying the presentation of conventional algorithms until children have had considerable experience at creating their own:

"If we challenge students with more opportunities for making estimates and mental computations and show them the conventional algorithms only after they have experienced a fertile period of inventing their own efficient procedures for solving problems like  $32 + 59$ , we can expect more young students to demonstrate the numeric partitioning flexibility that is characteristic of those with good number sense. If students then learn a conventional algorithm, they will view it as one of many ways to find a sum or difference; they'll be able to choose from and use, as appropriate, estimation, mental computation, and calculators, as well as invented and conventional algorithms" (Ross, 1989, p. 51).

"There has been a huge amount of evidence from our national testing (in England) that once given a calculation in vertical form, kids automatically go on to do it column by column and do not think about it" (Askew, 1999). The expectation in the National

Numeracy Project (England) is that by the age of about 9, 80% of all kids should be able to mentally add or subtract two two-digit numbers, and up to that point children should not be doing any vertical computations at all. Delay until they can add and subtract 2 two digit numbers in their heads.

Many people believe that there is no place for introducing conventional algorithms to children in the first three years of school. By giving arithmetic a problem solving focus, and by providing a whole range of problems for children to solve (preferably in story contexts of interest to children), we redefine the role of students in arithmetic, in the words of Lampert (1989), from the task of "*remembering* what to do and in what order to do it, to a problem of *figuring out* why arithmetic rules make sense in the first place" (p. 34).

Doug Clarke suggests using a variety of story problems, with increasingly large numbers, challenging children to solve them, by any method that makes sense to them. Through sharing their methods, children can make a start on the process of evaluating the various methods for their mathematical validity, their efficiency, and their generalisability, though not in these terms!

*Did you know?*

Results from the Learning Decimals project (Stacey & Steinle, 1995-1999) indicate that students' misconceptions about decimals are a significant problem. Data collected in this project show that approximately 45% of Year 9 students, and 40% of Year 10 students, were unable to compare decimals correctly. In the long term there is a general trend towards expertise, but as many as 40% of students may retain their misconceptions about decimals. "Misconceptions arise naturally, from students being unable to assemble all the relevant ideas together or from limited teaching, but students can easily be helped to expertise. Even a small amount of targeted teaching makes a difference." (Steinle, Stacey, & Chambers, 2002, CD ROM.

By the end of the project 75% of the students in this Year 6 class were using an expert strategy to compare decimals. This was a marked improvement from the 25% of students who were using an expert strategy at the beginning of the project. Additionally, the data from the project shows that the percentage of students using an expert strategy grew from 75% to 84% over the following year. This compares with 54% of Year 7 students who were found to be using an expert strategy in the Steinle and Stacey study, and is comparable to the improvement noted in the short intervention study by Helme and Stacey (2000).

These results indicate that students can make progress in their learning beyond what one would expect with normal growth and development, and furthermore, this learning can be retained over time. We were able to do this because we had accurately established the ways that the students were thinking about decimals, using the Decimals Comparison task, and we were then able to use this data to plan appropriate activities to challenge their thinking.

### Decimal Comparison Test

Moloney, J. & Stacey, K. (1996). Understanding Decimals. *The Australian Mathematics Teacher*, Vol. 52, no. 1, 4-8.

Complete the following task and glue this into your book.

Date:

On each line below there is a pair of decimal numbers. Put a ring around the larger one of the pair.

4.8	4.63
0.4	0.36
0.100	0.35
0.75	0.8
0.37	0.216
4.08	4.7
2.621	2.0687986
3.72	3.073
0.038	0.2
8.0525738	8.514
4.4502	4.45
0.457	0.4
17.353	17.35
8.24563	8.245
5.62	5.736

After you have completed this, try to explain what strategy you used to do this.

Table 2

Percentage of students using each strategy on the Decimal Comparison Test: Various samples

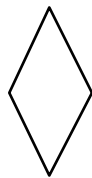
	Task Expert	Whole number strategy	Fraction strategy	Unclassified
<b>Project Class</b>				
Grade 6 ( $N = 29$ )	25	45	22	8
Pre-test (March, 2001)				
Post-test (August, 2001)	75	13	8	4
Grade 7 ( $N = 29$ )				
Delayed Post-test (November, 2002)	84	8	4	4
<b>Comparison Data</b>				
Steinle & Stacey (1998)				
No intervention				
Grade 6 ( $N = 319$ )	52	17	12	19
Grade 7 ( $N = 814$ )	54	13	14	19
Helme & Stacey (2000)				
Short intervention				
Pre-test Grade 6 ( $N = 47$ )	57	6	6	30
Post-test Grade 6 ( $N = 47$ )	81	6	0	13

1: Unclassified calculated from other figures

*Did you know?*

..that the language used by teachers in mathematics can be confusing for children?  
If you use MAB, what do you call each block?

What is the name of this shape? Diamond or rhombus?



Reflection:

- ◆ In what ways are children taught how to combine and separate quantities at your school?
- ◆ How many different strategies do each of the children you work with have to work things out?
- ◆ What percentage of children in your school have an accurate understanding of decimals?

- ◆ How consistent is the language of mathematics being used in the school?

## ***Teacher's values***

What was I valuing?

Prior to the dot cards exercise I was valuing the "getting of the right answer" to the exclusion of all else. I had given lip service to valuing the process as well as the product.

Now I value the development of Relational understanding. This is the process of connecting mathematical concepts and relationships (conceptual knowledge) with the symbols, rules and procedures that are used to represent and work with mathematics (procedural knowledge). Relational Understanding is evident when the rules or procedures of mathematics have a conceptual and meaningful basis and the concepts can be represented by appropriate symbolism.

Relational Understanding is evident when we can draw on the mathematical knowledge that we need when we need it and use it to solve any problems we encounter and use it to communicate with others.

Understanding is "what students need for next week and the week after, next year and the year after. It is sustainable learning. It is the only kind worth spending their time on - anything less than understanding isn't worth the risk" (Willis, 2000, p. 33).

I have come to appreciate the great diversity in learners and I am very interested in trying to understand how they are thinking. This can give me great insight and help me plan appropriate learning activities for each learner. Through the use of carefully constructed open ended activities children can enter and exit at different points and feel successful as learners.

## ***About mathematics and numeracy***

A major policy objective of the Commonwealth Government is to ensure that all students attain sound foundations in literacy and numeracy. In 1997 all Education Ministers agreed to a National Literacy and Numeracy Plan that provides a coherent framework for achieving improvement in student literacy and numeracy outcomes.

The 1999 Adelaide Declaration on National Goals for Schooling in the Twenty-First Century states that in terms of curriculum, students should have:

2.1 *Attained high standards of knowledge, skills and understanding through a comprehensive and balanced curriculum in the compulsory years of schooling encompassing the agreed eight key learning areas (of which mathematics is one) and the interrelationships between them*

2.2 *attained the skills of numeracy and English literacy; such that every student should be numerate, able to read, write, spell and communicate at an appropriate level*

**We need to be very clear about what Numeracy is, what Mathematics is and the relationship between the two.**

**We need to make time to clarify these ideas with staff, parents and children, and to make explicit what we do and why we do it in the light of these perspectives.**

Through my most recent work with MLATS in the Kindergarten, we have adopted the following descriptions of mathematics and numeracy.

***Numeracy*** can be described as the “**intelligent, practical use of *mathematics* in context**” (Sue Willis)

***Mathematics*** can be described as all of the **concepts, the procedures, the language, the strategies and the skills** which relate to **measurement, shape and structure, pattern, data, number and chance**”.

Numeracy and Mathematics are not the same. Numeracy goes beyond mathematics and draws on all of the mathematical knowledge one has in real contexts.

**Success in mathematics** is evident when a learner can draw on their knowledge when they need it.

The development of numeracy is “everyone’s business” (AAMT).

Reflection:

To what extent do the children in your school have opportunity for the “intelligent, practical use of mathematics in context”?

In these examples, I include pre-schools when using the word schools.

For example,

- ◆ some schools have student forums where children form groups according to interest and need in their schools. They are given a budget and work towards

achieving goals for their school. This might be the development of a play space, a quiet space, lunchtime games and activities, and so on. Teachers are involved in this process as part of the group, not as the leader.

- ◆ Other schools plan units of enquiry and through these extended periods of time children draw on a wide range of mathematical ideas and skills to enhance their learning.
- ◆ Other schools have children involved in the running of the school canteen or in the school office or the uniform shop, even in the formation of the school timetable.

These are real working situations where children's experiences will be enhanced by the extent to which they can draw on a range of mathematical ideas when they need them.

When the National Numeracy Goals and sub goals were first formulated a National action plan was also developed.

Two of the key points of this action plan were that teachers

- ◆ Identify children at risk of not meeting the national Numeracy benchmarks and they
- ◆ Intervene as soon as possible to support them in their learning

Why do some children fail?

- ◆ More likely than not, there is a mismatch between current knowledge of the learner and the curriculum which is being taught.

How do we measure student's mathematical knowledge, skills and understandings in a socially just way? How do we measure their numeracy? These are quite different things and cannot be measured in the same way.

Many students appear to know more than they actually do because they are able to give the correct response or perform a calculation correctly by relying on rules they do not understand (Thomas, 1996; Steinle & Stacey, 1998). How do we accurately establish what they do know?

Already talked about Decimals Comparison Task.

**Early Years Numeracy Interview data.**

*(Reference: Ann Gervasoni, 2000, Using Growth Points to Identify Year 1 Students who are at Risk of not learning School Mathematics Successfully, Australian Catholic University, MERGA, July 2000.)*

Children were found to be at 5 levels of risk (5 being at the most risk) at the beginning of year 1.

## OHT

### Risk Level 5

Children who cannot yet

- count a collection of 20 objects
- read, write, order or interpret one digit numbers, or
- count on.

### Risk Level 4

Children who can count a collection of about 20 but cannot yet

- read, write, order and interpret, one digit numbers, nor
- count on

### Risk Level 3

Children who can count a collection of about 20 items, but not yet

- read, write, order and interpret two-digit numerals, nor
- count on

### Risk Level 2

All other children who cannot yet count on

### Risk Level 1

All other children who cannot yet count all in multiplicative situations

## First Steps (WA)

### Example



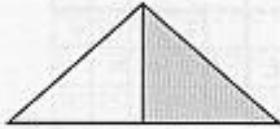


### Did You Know?

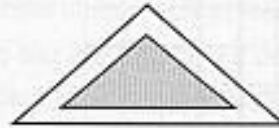
A diagnostic activity for the middle and later years

Pose the following problem.

Kia and Sam had different types of sandwiches. They decided to give each other exactly half a sandwich. The picture below shows how they cut their sandwiches into two pieces.



Kia's sandwich



Sam's sandwich

Sam took the shaded part of Kia's sandwich, but Kia wasn't sure that she could take exactly half of Sam's. What should she do?

Ask: Do you think that the shaded piece of Sam's sandwich could be a half? If you think it is possible, can you think of a way to check?

Many students initially think that halves have to match exactly; that is, look the same. Many students would not accept that the shaded shapes could be half the area of the triangle. Others—usually younger students—may think that any two parts is half and so would not bother to check at all.



#### Intervention:

Current project is working on an intervention project with a small groups of teachers - teachers have used the program in different ways - early results are encouraging.

"Even a small amount of targeted teaching can make a difference"

Regular, explicit, "connective" experiences, and parent support and expectations may be very important here.

#### Reflection:

- ◆ What procedures do you have at the school level to (a) reduce risk in numeracy development (b) intervene to provide help when it is needed?
- ◆ What procedures do we have in place at our school to ensure that you are helping ALL children learn mathematics in such a way that they will be able to draw on it when they need it?
- ◆ What commitment in terms of resourcing (staffing, materials) do you give to numeracy? How does this compare to your commitment to literacy?

- ◆ If a child is "failing" in school mathematics whom do you blame? What excuses do you make? What expectations do you have for this child?

### **To conclude:**

Brain based Learning emerged in the 1980's- a new way of thinking about learning - if you want to maximise learning you will need to discover how the brain works.

This has fuelled a massive and urgent world wide movement to redesign learning.

What we thought was critical in the past may not be very important at all. You may have the most efficient net in the world but if you are fishing in the wrong place you are still not going to go home with a big catch. Many traditional teaching approaches ignore individual's life circumstances and therefore the needs of the brain. P 14

The brain is designed for survival not formal instruction, in fact "The brain is poorly designed for formal instruction" p 3 The brain will choose or select the learning that will best enhance their chances of survival The brain will only concentrate on instruction if it is perceived as meaningful and only if the brain's primary survival needs have been satisfied p 14 The brain is trying to learn in order to survive, this is why we need to centre our approaches on the needs of the learner. The learner's own goals, when encouraged to be identified may include social, economic and personal considerations that you were unaware were important to this individual.

We have vastly underestimated the capacity of learners - keep in mind that the children you teach are capable of learning beyond your wildest dreams

### **Brain based learning tells us that**

- ◆ Complex multi-path, multi-modal, multidisciplinary learning is natural (the brain simultaneously operates on many levels of consciousness, processing all at once a world of colours, movements, emotions, shapes, smells, sounds, tastes, feelings and more. It assembles patterns, composes meaning, and sorts daily experiences from an extraordinary number of clues. Simultaneously the brain is attaching emotions to each event, If we teach in a linear, structured and predictable fashion we unknowingly inhibit the brain's learning ability and the result of this is bored or frustrated learners who then perpetuate the underachievement cycle and who act out in negative or disruptive ways)
- ◆ We understand complex topics better when we experience them with rich sensory input, as opposed to merely reading about them or hearing about them. As children we learn about our world from scattered, random input that was messy at times and that left room for exploration and manipulation. As

children, most of what was imprinted on our brains was done in a fairly chaotic way as we figured things out through trial and error.

- ◆ The more we know about the brain and how it works the better decisions we make about learners and learning; the more often we can reach more learners

Take this home with you:

**It is CRUCIAL that I seek to accurately establish what children know and build on from this knowledge.**

**I need to seek new ways to measure what is important rather than think that what can be measured is important.**